MATH 321 Final Examination

Your name and surname: Jan 16, 2001
Your nickname (if applicable): 9:00–12:00
Your best signature: PARK 1

Question	Score		5	/10 points		10	/5 points
1	/5 points		6	/10 points		11	/5 points
2	/5 points		7	/10 points		12	/10 points
3	/5 points		8	/7 points		13	/15 points
4	/5 points		9	/8 points		Total	/100 points

(1) Give an example of a fact that you have known long before taking MATH 321 but which you recognized in MATH 321 to be a piece of information about groups. (5 points)

(2) Denote the residue class of $a \in \mathbb{Z}$ modulo 18 by $\bar{a}$, modulo 24 by $\tilde{a}$. Is the "mapping" $\varphi : \mathbb{Z}_{18} \longrightarrow \mathbb{Z}_{24}$, $\bar{a} \mapsto \tilde{a}$ a group homomorphism? (5 points)

(3) Are $\mathbb{Z}_9^\times$ and $\mathbb{Z}_{18}^\times$ isomorphic groups? (5 points)

(4) Can there exist a group G and an element a of G such that $o(a^3) = 20$ and $o(a^5) = 9$? Either give an example of such a pair G, a, or prove that this is impossible. (5 points)

(5) Let $F = \langle f \rangle$ be a cyclic group of order 4 and $T = \langle t \rangle$ a cyclic group of order 10. Find all group homomorphisms from F into T. (10 points)

(6) If a, b, $c \in \mathbb{Z}_{11}$ and $G := \{g \in \mathrm{GL}(2, \mathbb{Z}_{11}) : \det g \in \{4, 5, a, b, c\}\}$ is a group, what are a, b and c? (10 points)

(7) Let G be a group, H a commutative group, $\varphi : G \to H$ a homomorphism onto H. Let N be a subgroup of G that contains $\operatorname{Ker} \varphi$. Show that N is a normal subgroup of G. (10 points)

(8) Let H be a subgroup of a group G. Suppose $g \in G$ and $o(g) = n \in \mathbb{N}$. If $g^m \in H$ and if m and n are relatively prime, prove that $g \in H$. (7 points)

(9) Let G be a group, $N \leq G$, $A \subseteq G$. If $N \trianglelefteq G$, does $AN = NA$ have to hold true? (8 points)

(10) Give an example of a field different from $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}_p$ (p prime).

(5 points)

(11) Characterize all rings in which the identity $x^2 - y^2 = (x + y)(x - y)$ is valid.

(5 points)

(12) Let A be an ideal of R. Is it true that $\mathrm{Mat}_2(R)/\mathrm{Mat}_2(A) \cong \mathrm{Mat}_2(R/A)$?
(10 points)

(13) Let D be a principal ideal domain, a, b relatively prime elements of D. Is D/abD isomorphic to $D/aD \oplus D/bD$? (15 points)