

**Math 322 First Midterm Examination**

Name : \_\_\_\_\_

April 8, 1999

Signature : \_\_\_\_\_

9:00–10:30

(1) Is  $3x^6 - 4x + 6$  irreducible in  $\mathbb{Z}[x]$ ?

(10 points)

(2) Does there exist a polynomial  $f$  in  $\mathbb{Z}[x]$  such that  $f(0) = 1, f(1) = 2, f(2) = 3,$   
 $f(3) = -2$ ? (20 points)

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(3) For  $(a, b), (c, d) \in \mathbb{Q} \times \mathbb{Q}$  and  $a \in \mathbb{Q}$ , we put  $(a, b) + (c, d) = (a + c, 0)$  and  $a \cdot (c, d) = (ac, ad)$ . Do these operations turn  $\mathbb{Q} \times \mathbb{Q}$  into a vector space over  $\mathbb{Q}$ ?  
(20 points)

(4) Find a field  $K$  and a vector space  $V$  over  $K$  such that  $V$  has exactly 4 elements.  
(20 points)

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(5) Why is the notion of dimension important?

(30 points)