Math 322 Second Midterm Examination

Name: _____ May 18, 1999

Signature: _____ 9:30–11:00

(1) Is $\mathbb{Q}(\sqrt{3}+\sqrt{5}+\sqrt{7}) = \mathbb{Q}(\sqrt{3},\sqrt{5},\sqrt{7})$? Why or why not? (10 points)

(2) Find $|\mathbb{Q}(\sqrt{3}+\sqrt{5}) : \mathbb{Q}(\sqrt{3})|$. Justify your answer. (15 points)

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(3) Find the number of irreducible monic polynomials of degree 36 in $\mathbb{F}_3[x]$, and in $\mathbb{F}_5[x]$. (15 points)

(4) Construct a field in which $x^2 + x + 1 \in \mathbb{F}_5[x]$ has a root. Describe its elements, explain how addition and multiplication are carried out and how inverses of nonzero elements are found. (20 points)

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(5) Let A, B be fields and let C be a division ring such that $A \subseteq B \subseteq C$, so that C is a vector space over A and over B, and B is a vector space over A. Suppose that $\dim_A B = s \in \mathbb{N}$ and $\dim_B C = r \in \mathbb{N}$. Is it true that $\dim_A C = rs$?

(20 points)

(6) Let K, E be fields and let D be an integral domain such that $K \subseteq D \subseteq E$. If E is algebraic over K, does it necessarily follow that D is a field? (20 points)