

MATH 322 Final Examination

Name:
Surname:
Signature:

June 2, 2001
9:00–12:00
TB 120

Question	Score		5	/5 points		10	/20 points
1	/10 points		6	/5 points		Bonus Question	/20 points
2	/15 points		7	/10 points			
3	/5 points		8	/15 points			
4	/5 points		9	/10 points		Total	/100 points

1. Express the symmetric polynomial $x^3 + y^3 + z^3 + xyz$ over \mathbb{Z} in terms of the elementary symmetric polynomials. (10 points)

2. Let

$$A := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{Mat}_{2 \times 2}(\mathbb{C}) : \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\} \\ \subseteq \text{Mat}_{2 \times 2}(\mathbb{C}).$$

Is A a vector space over \mathbb{R} ? Over \mathbb{C} ?

(15 points)

3. Let K be a field. Give the definition of *the characteristic of K* . (5 points)

4. Let K be a field of characteristic $p \neq 0$. Prove that $\varphi : K \longrightarrow K, a \longmapsto a^p$ is a field homomorphism. (5 points)

5. Find the multiplicative inverse of $2 + 3\sqrt{5}$ in the field $\mathbb{F}_7(\sqrt{5})$. (5 points)

6. Let E/K be a field extension and let D be an integral domain such that $K \subseteq D \subseteq E$. Prove that, if E is algebraic over K , then D is a field. (5 points)

7. Find the number of monic irreducible polynomials of degree 72 over \mathbb{F}_5 .
(10 points)

8. Let E/K be a field extension and let L be an intermediate field. If L is (K, E) -stable, prove that L'' is (K, E) -stable, too. (15 points)

9. Find an algebraic field extension that is not separable.

(10 points)

10. Let $\zeta := e^{\frac{2\pi i}{12}} \in \mathbb{C}$ and $E := \mathbb{Q}(\zeta)$. Find all subgroups of $G := \text{Aut}_{\mathbb{Q}}E$, all intermediate fields of E/\mathbb{Q} , and a primitive element for each of the intermediate fields. Describe the Galois correspondence by Hasse diagrams of groups and intermediate fields. (20 points)

Bonus question. Tell about the contribution to algebra of one of the following mathematicians: Gauss, Cauchy, Abel, Liouville, Klein, Kronecker, Dedekind, Heinrich Weber, Artin.