(1) Is $3x^6 - 4x + 6$ irreducible in $\mathbb{Z}[x]$? (10 points)
(2) Does there exist a polynomial $f$ in $\mathbb{Z}[x]$ such that $f(0) = 1$, $f(1) = 2$, $f(2) = 3$, $f(3) = -2$? (20 points)
(3) For \((a, b), (c, d) \in \mathbb{Q} \times \mathbb{Q}\) and \(a \in \mathbb{Q}\), we put \((a, b) + (c, d) = (a + c, 0)\) and \(a \cdot (c, d) = (ac, ad)\). Do these operations turn \(\mathbb{Q} \times \mathbb{Q}\) into a vector space over \(\mathbb{Q}\)?

(20 points)
(4) Find a field $K$ and a vector space $V$ over $K$ such that $V$ has exactly 4 elements. 
(20 points)
(5) Why is the notion of dimension important? (30 points)