1. Express the symmetric polynomial \( x^3 + y^3 + z^3 + xyz \) over \( \mathbb{Z} \) in terms of the elementary symmetric polynomials. (10 points)
2. Let

\[
A := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{Mat}_{2 \times 2}(\mathbb{C}) : \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\} \\
\subseteq \text{Mat}_{2 \times 2}(\mathbb{C}).
\]

Is \( A \) a vector space over \( \mathbb{R} \)? Over \( \mathbb{C} \)?

(15 points)
3. Let $K$ be a field. Give the definition of the characteristic of $K$. (5 points)
4. Let $K$ be a field of characteristic $p \neq 0$. Prove that $\varphi : K \rightarrow K, a \mapsto a^p$ is a field homomorphism. (5 points)
5. Find the multiplicative inverse of $2 + 3\sqrt{5}$ in the field $\mathbb{F}_7(\sqrt{5})$. (5 points)
6. Let $E/K$ be a field extension and let $D$ be an integral domain such that $K \subseteq D \subseteq E$. Prove that, if $E$ is algebraic over $K$, then $D$ is a field. (5 points)
7. Find the number of monic irreducible polynomials of degree 72 over $\mathbb{F}_5$. 

(10 points)
8. Let \( E/K \) be a field extension and let \( L \) be an intermediate field. If \( L \) is \((K, E)\)-stable, prove that \( L'' \) is \((K, E)\)-stable, too. (15 points)
9. Find an algebraic field extension that is not separable. (10 points)
10. Let \( \zeta := e^{\frac{2\pi i}{12}} \in \mathbb{C} \) and \( E := \mathbb{Q}(\zeta) \). Find all subgroups of \( G := \text{Aut}_\mathbb{Q} E \), all intermediate fields of \( E/\mathbb{Q} \), and a primitive element for each of the intermediate fields. Describe the Galois correspondence by Hasse diagrams of groups and intermediate fields. (20 points)
**Bonus question.** Tell about the contribution to algebra of one of the following mathematicians: Gauss, Cauchy, Abel, Liouville, Klein, Kronecker, Dedekind, Heinrich Weber, Artin.